

# **Advanced Modeling for Holding Company Valuation**

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How widely varying degrees of fact patterns and control can be represented with conventional and advanced valuation models, including present value, put option, short-term liquidating partnerships, restricted stock studies and other proxies. Techniques can be applied for any asset holding circumstance, including those annoying general partnerships and cotenancies. Principles can also be used to access any degree of complexity presented by tiered entities.

## ***Introduction***

Asset holding companies are a special case—a narrow slice of the business valuation domain. Sometimes simplifying, sometimes confounding, always entertaining (or annoying, depending on your point of view).

The holding company valuation is largely defined based on distribution of control. Organizational types in order of increasing control generally attributable to the interest holder are:

- Limited partnership/LLC
- Corporation
- General partnership
- Common tenancy (real property)

The first two are well understood, and the valuation process is straightforward (although the role of the asset in the valuation is not always fully considered, an issue that will be addressed throughout this paper). General partnership and cotenancy interest valuation is unsettled, both because the interests normally have more control, and because asset characteristics are tightly woven into all the facts & circumstances that bear on value.

So what is a general partnership, anyway? It can be treated as an LP or LLC, but its general partners normally have at least some control. Value has a great deal to do with the nature of the asset, but the asset is not directly controlled by the partner—there is usually *something* or *someone* in the way of its exercise of basic property rights. Can we treat it as an asset holding company, normalize the statements and adjust discounts for attributes of control? Do we start at the minority level, or at the asset level of value? What sort of data is relevant? Can conventional valuation models be applied? What about the simplest form of undivided asset ownership—common tenancy? How is it different? How is it the same? Is cotenancy an asset valuation problem, a business valuation, or both? “Special Case, or Business as Usual?”

It would seem strange that advanced valuation models would be applicable, considering the relatively simple nature of the valuation problem. However, “simple” direct comparison is anything but, and complexities can creep in making comparisons impossible; e.g. it is not unusual to see multiple structures layered upon one another, in which case “simple” no longer applies. Many models can provide useful depictions of market behavior, and useful indications of value *if* their limitations are understood, and they are carefully connected with the facts of the case.

## **The key premises of this paper are**

1. The nature of the asset(s) underlies the fact patterns.

The nature of the asset is tied directly to the motivation of the prospective buyer, and its specific characteristics are an essential part of the buyer's due diligence.

2. Risk and holding period are fundamental interpretations of the facts.

The foundation of the net asset value method is that the interest holder expects to realize the value of its pro rata share at some point in time. Investment in real property assets is, by its nature, connected with time. This is a fundamental departure from operating company valuation, where entity life is usually indefinite.

3. The valuation's starting point is the asset level of value.

Regardless of method, the valuation's reference is to the entity/asset level, not the minority level. The base data (values, growth rates, yields) come from asset markets, with only discount influences being extracted from minority-level transactions.

Any assignment regarding an undivided interest in an asset—a general partner, cotenant, limited partner or any other such position—is a multidisciplinary assignment.

## **Objectives**

- Be able to identify the basis for developing holding company fact patterns.
- Understand applicability and limitations of advanced discounting models.
- Interpret and reconcile the models' discount indications.
- Take home tools applicable for any complex asset-holding company structure.

This paper assumes the reader has a working familiarity with control and marketability/liquidity discounting models.

## ***Fact Application in the Valuation Process***

Fact patterns do not fall under the Advanced Modeling heading. However, they are absolutely fundamental to the effective use of any valuation model, and are so often overlooked for asset companies, that at least a mention in this paper is warranted. Usable detail is offered in a previous paper.<sup>1</sup>

Facts & circumstances are classified under the following headings: Assets – real property/balance sheet items; Owners – those that come with the deal; Ownership Structure – agreements, legal environment, any de facto realities; and Financial Structure – capital structure/asset financing. Elements under these headings are most readily expressed in asset values (*balance sheet items*), *cash flows*, growth rates, *risk-adjusted yield rates*, and, of course, *holding period*.

Balance sheet items include FMV of assets and liabilities and terminal value (reversion) of NAV. Projecting the balance is necessary for discounted future returns and partition methods of valuing the discount.

Cash flows used in discount analysis are usually based on the appraiser's asset operating forecast, adjusted for non market conditions (the difference between the appraiser's transfer hypothesis and continued operation by the subject entity), future events (such as lease turnover or explicit capital replacements) that may require large expenditures, and others. In the event that early disposition of assets is postulated, then any loan prepayment penalty may have a large effect on value.

Growth rates are largely a property of the underlying asset, and its associated leases, financing and market conditions.<sup>2</sup> Expectations for value and cash flow growth are explicit variables in income methods, and important points of comparison with other types of market data.

Risk associated with the interest begins with the market risk represented by the assets. Expression as an asset-level yield rate allows us to build up an impaired-level yield with premia added for specific circumstances that create uncertainties. Additional facts & circumstances point toward selection of a likely holding period, over which the risks are experienced...

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<sup>1</sup> A shopping list of real property company facts are included in "Integrating Real Property & Business Valuations," presented at the ASA International Conference, July 2007.

<sup>2</sup> Long-term growth may be affected by temporary nonmarket lease conditions, or future tenancy changes. Loss of a favorable loan could require forecasting loan terms and fees, and adversely affect cash flows. Equity build-up contributes to increased NAV. Expected changes in market conditions will likely affect value growth, but may or may not affect cash flow growth, depending on the willingness of the partners to make investments or otherwise take advantage of changes.

## ***Holding Period***

If we accept that an objective of the holder of an interest in an asset holding company, or any other undivided interest in an asset, would be to realize its pro rata share of net asset value at some future point, then time becomes an important and even critical element of the analysis. This is a fundamental departure from operating company valuation, where entity life is usually indefinite, and the buyer's expectations are entirely different.

Real estate assets in particular are, by their nature, connected with time cycles, and the market for such assets uses time as an important element of the investor's model. It is often related to motivations for selling as well. Thus, any process which attempts to model the market must take time into account in some fashion.

An issue may be whether the hold is discretionary (as it would be for the fee interest in a property), or forced for a period (by the ownership structure and circumstances). The choices available to the undivided interest holder are slim indeed, and its options for selling the interest before sale of the asset are typically either impossible or carry a severe penalty. Besides, sale at FMV (for a marketable interest) would eliminate most of the expected gain, and many methods of forcing sale carry their own risks, as noted earlier. The interest is not *marketable* in any meaningful sense of the word, but the fact that it is *illiquid* as a result of the forced hold is the most important. A better term for *hold* (which is generally discretionary) would be *blocked*.

The model period may be known, but the restricted period is usually not. The facts may point to a relatively certain exit point (say, the end of a partnership's term), or there may be many possible exits, carrying differing probabilities. Fortunately, many facts & circumstances associated with undivided interests and holding companies have something to do with time:

### **Real Property Assets**

- Lease turnover
- Highest & best use differences/changes
- Possible future development

### **Owners**

- Manager age/actuarial life
- Willingness to obstruct

## **Ownership Structure**

- Remaining term
- Vote to change/amend agreement
- Ability to withdraw

## **Financial Structure**

- Loan term/balloon payment
- Mortgage loan prepayment penalty

We might want to add the one-year capital gain period to the list. By itself, this would be a hold by choice, although the tax penalty on gain on an early sale arguably creates a de facto restriction.

If multiple facts & circumstances, which reveal themselves in the buyer's due diligence, point to specific exits, might the buyer not give them some consideration in its value modeling? Our task is to emulate the buyer's process, which is just as well, since most available valuation models—comparative company, options and present value models—all observe market response to a restricted time element in some fashion. The market for the underlying assets also suggests the buyer's model period.

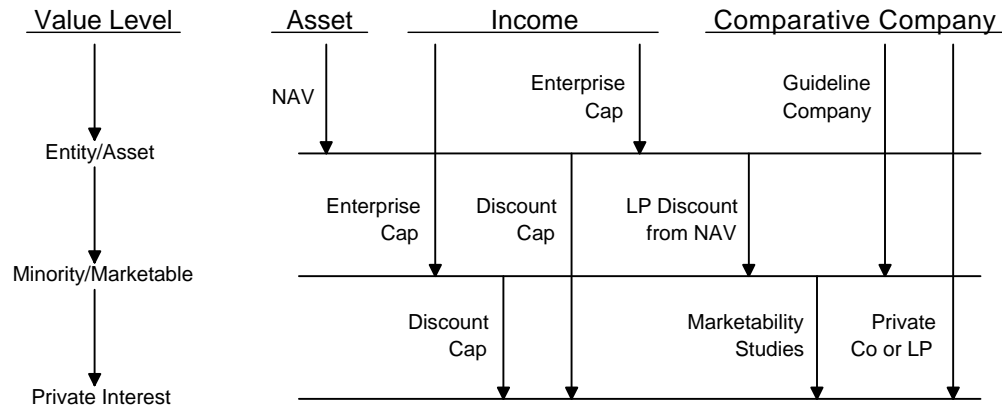
Time and risk are quantitative expressions of the facts & circumstances of the case, and their use in a valuation model allows us to mirror the buyer's process, and to understand pricing sensitivity to variations in the assumptions—a major due-diligence benefit. Once the inputs are established, valuation models can be selected based on how well they match the selected variables—does the model apply for the range of variables selected, and is it a reasonable representation of likely market participant behavior?

Uncertainty is the name of the game to some extent anyway. There is no point in pretending we can have an unequivocal set of facts to work with—no buyer does, so why fret? We can find reasonable quantitative expressions of various facts to use with demonstrably valid models, to produce an adequately reliable and supported opinion of value, as we shall see. Who could ask for more?

## Valuation Process Overview

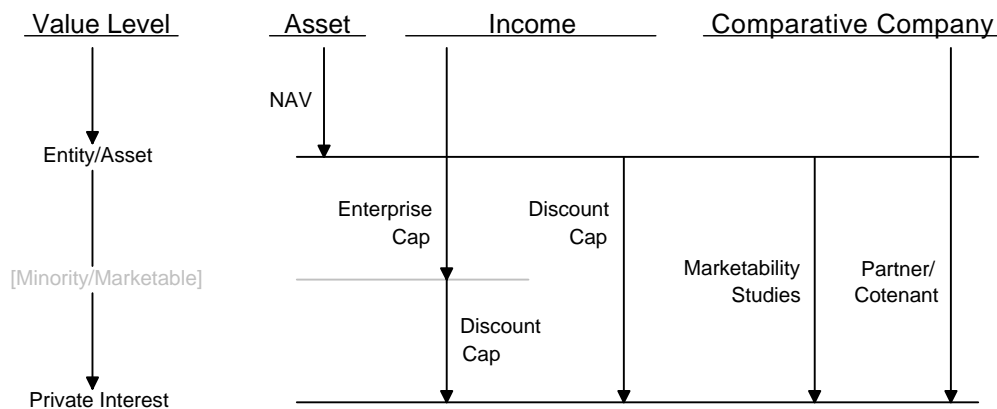
Numerous valuation methods are available for use with asset holding companies. Figure 1 shows the relationship of various methods to the traditional levels-of-value hierarchy. Facts are incorporated in each method, and many end up influencing value at multiple levels.

**Figure 1 - Asset Holding Co Valuation Methods**



The situation is simplified for general partnerships and cotenancies, since the minority/marketable level is not directly usable. However, the expectation of a pro rata share of NAV at the end of the holding period allows us to apply discounting methods directly to the asset level of value. Several holding company methods are eliminated, as shown in Figure 2.

**Figure 2 - General Partner and Cotenancy Valuation Methods**



Income methods can produce values at any level. In this case, the base rates come from the assets, and added risk premia can extend all the way to the private interest level. The minority-marketable value can be computed, but a partial control premium would have to be applied for the level to be meaningful—but it still wouldn't be useful, as there is no

direct comparison for partial control. Guideline company is a broad methodological category, and for this purpose can include direct comparisons to sales of similar interests, restricted stock methods, and a specific class of public limited partnership transaction.

## Valuation Models

Now we know something about facts, have adjusted the balance sheet, and prepared a cash flow forecast. How do they influence any discounts that might be attributable to control and liquidity impairments? What methods do we have to track investor thinking, either in the market for the subject interest (whatever that is), or in proxy markets? Can we extract meaningful influences from these other markets that connect with the facts?

The benefit of having developed quantitative expressions for a large portion of the facts is that we have points of comparison that can be used with many valuation models: Single- and multi-period capitalization, options, direct sales, restricted stock studies, and some partnership transactions.

Valuation models are all trying to achieve the same goal—to reveal the decision-making process of buyer and seller. There is no point in trying to establish one as “more valid” or superior to another in some general way, as their applicability and reliability will be determined based on how well they fit the specific case. All provide transfer functions that we can apply to convert our selected variables into values or discounts. They contain market data or are developed using market observations, and also contain boundary conditions and certain inherent assumptions. Using multiple valuation models can significantly increase the reliability of the value conclusion, since in many cases, we have little understanding of the market we are interpreting, and looking at it from several angles is extremely helpful.

General partnerships and cotenancies (and to a lesser extent all asset holding companies) have the benefit of simplicity. Therefore, much of the debate concerning levels of value and applicability of specific valuation models can be avoided. For example, liquidity vs. marketability concepts are moot, since liquidity deals with transfer of the interest as a method of taking out its holder, while we are only concerned with the fact that it is difficult (or impossible), and that its holder will hang in there until asset liquidation.

Given this, marketability and liquidity terms may be inappropriate, because of the mixed connotation. We are after market observations and market-based models that address the cost of risk, exacerbated by an inability to exit the investment, as a function of the value to be received at exit—the discount from pro rata NAV. Of course, the market observations we might use, from studies, transaction data or options functions, *are* measuring marketability and illiquidity...

Available valuation models include:

***Income methods***

- Partition time and cost (“cost-to-partition”)
- Constant growth/present value
- Discounted future returns

*Income methods are fundamental, as risk and holding period are basic points of comparison for nearly all applicable methods*

***Comparative company methods***

- Direct sales of fractional interests  
Includes both individual transactions and studies
- Guideline partnerships - Short-term liquidating
- Proxy data – restricted stock studies

***Other methods***

- Options trading models

## Income Methods

Income methods are handy because they directly incorporate fact-related elements identified above. All income models make use of cash flows, reversion, growth, yields and holding period. They can be a direct representation of the investor's alternatives (depending on how the yield rate is developed), and have the advantage of directly incorporating the quantitative expressions from analysis of the facts & circumstances. However, they don't always work... (for short or very long periods; see comments below).

### Yield rate development – asset level

The starting point for rate development is the asset. The market for assets rarely uses a build-up method beginning with a risk-free rate, but we are provided with copious and detailed survey data,<sup>3</sup> and/or the capitalization or discount rate used by the real estate appraiser.<sup>4</sup> The asset represents much of the risk, and asset rates are a stable and reliable starting point.

The asset-level rate is an equity yield, which includes any balance sheet effects. As the capital structure of most real estate holding companies is fixed (or only changed on rare occasion), the presence of a declining-balance mortgage loan *requires* the use of a multi-period model. The yield can be calculated simply as the internal rate of return (IRR)<sup>5</sup> to equity NAV<sup>6</sup> over the selected holding period. Equity growth rates can also be calculated directly, value growth from NAV to distributable proceeds on sale, and cash flow growth from year 1 to year n.

### Yield rate development – attributes associated with control

The asset-level yield rate represents risk associated with direct control over the assets, where all property rights are intact. Any control limitations represent fewer rights and increased risk, and the liquidity restrictions remove the right to exit until the end of the term.

The control risk premium can be approached by beginning with the minority rates published by Partnership Profiles, in their Rate of Return Study.<sup>7</sup> The Study provides

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<sup>3</sup> The Korpacz Investor Survey, various surveys at RealtyRates.com, and other published survey data present rates, growth and other detailed property market data.

<sup>4</sup> Proxy rates can be developed for non income-producing property types.

<sup>5</sup> Use of IRR mirrors market practice for real property assets.

<sup>6</sup> In cases where there is no independent analysis of NAV, the asset rate can be extracted from the market, and NAV calculated as the present value of future cash flows. This is useful when there is no asset appraisal and NAV is needed for disclosure or other purposes.

<sup>7</sup> Partnership Profiles, Inc., 2007 Rate of Return Study, Publicly-Held Real Estate Limited Partnerships and Real Estate Investment Trusts, (2007, Partnership Profiles, Inc., Dallas, TX). The study is also excerpted in: Bruce A. Johnson, ASA, Spencer J. Jefferies and James R. Park, ASA, Comprehensive Guide For The

with minority level risks for an “average” partnership (within broad classifications). The real property/asset risk can be removed from this yield rate, by subtracting a broad-based real property rate, such as the Korpacz Yield Indicator (KYI).

Yield from ROR Study	18.7%
Remove RP influence (KYI)	-11.2%
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Noncontrolling risk premium	7.5%

The partnership risk premium is the remainder—the risk premium due to the partnership structure, at the noncontrolling level of value. However, a typical general partner or cotenancy interest usually has some attributes of control, or at least an ability to block undesired control by others. To the extent that these attributes have not already been captured in the cash flow or balance sheet adjustments, they need to be added back, reducing the risk premium.

There may also be other control-influenced risks, or differences between the subject entity structure/characteristics and the public limited partnerships from which this premium was derived. Leverage can increase risk, as demonstrated in the partnership data, and a further premium adjustment can be extracted from the Rate of Return Study. On the other hand, if the interest has the ability to change the partnership’s capital structure, any such risk may be reduced. Partnership yield premia are also affected by the distribution rates, as limited partners will accept a lower total return if cash flows are greater. Other variables such as size and diversification may also need to be adjusted, although the support for these is limited.<sup>8</sup> Another matter is the relative illiquidity of the secondary market. This is normally a small adjustment, whose effect is not settled.<sup>9</sup> It would be more important if we were trying to achieve a true minority-marketable value, but its effect may be swamped by the other uncertainties present for general partner and cotenancy interests.

Consider a 3-partner general partnership, where the subject is not the managing partner, but his consent is needed for major decisions (borrow or lend funds, sell assets, make major capital investments), but not for operational reasons (leasing, property management). There is some ability to protect against perceived adverse decisions, but the major attribute of control, ability to sell the assets, is not present. Also assume low leverage, and a current distribution rate of 5.8%, slightly greater than average for the public limited partnership database.

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Valuation of Family Limited Partnerships, 3rd Ed. (2003, Partnership Profiles, Inc., Dallas): pages 18-21.

<sup>8</sup> Dennis A. Webb, ASA, MAI, MRICS, *Valuing Undivided Interests in Real Property—Partnerships and Cotenancies*, (Chicago: Appraisal Institute, 2004): 94-95. The Partnership Profiles data demonstrates a strong effect on discount (and total yield) from the distribution rate, but also that the measurable influence of both size and diversification is weak at best.

<sup>9</sup> Ibid. See also John, Jeffries and Park, *Supra.*, pages 32-34.

The equity yield, adjusted for these attributes of control, is then:

Subject equity yield	9.1%
Partnership risk premium	7.5%
Adjust for distribution rate differential	-0.5%
Adjust for attributes of control	-3.5%
Adjust for secondary market illiquidity	-0.5%
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Adjusted equity risk (control)	12.1%

This is not exactly a minority-marketable level rate.<sup>10</sup> Within the income method, it is just as well to keep going, and adjust further for facts & circumstances that exacerbate this risk during the “forced” hold.

#### Yield rate development – attributes associated with illiquidity

Let us further assume that the facts point to a five-year hold expectation of the hypothetical buyer. Of course, this is not certain, and hold period uncertainty increases risk. It is easy to test sensitivity of the model; say, a one-year change in holding period would be equivalent to changing the yield rate by 1.1%. Depending on the degree of uncertainty, the rate might be adjusted from, say, 0.5% to 2.0%. This is just backing into a probabilistic notion of holding period, and a more rigorous result could be achieved by calculating values using multiple holding periods, and weighting the results.

Smaller interests have difficulty absorbing due diligence costs, which are affected by complexity, confidentiality provisions, refusal rights, and property risks. Adjustment can be based on actual costs (over those implicitly included in the property appraisal) if feasible. If adequate due diligence is not performed, then there is little or no cost, but risk is increased.

Business/management risk is normally a factor when there is some chance that operating the real estate will become a problem due to differences of opinion over management issues, or when management competence (in relationship to the real property market and public limited partnerships) is questionable. The property in our case has a long-term, triple-net lease, so few, if any, management decisions are required. However, this would change if the tenant were to decline to renew in year five (the terminal year), and a decision to sell or redevelop were at issue. Such decisions require unanimity, and a small adjustment is required.

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<sup>10</sup>It could conceivably be a starting point for applying marketability discount data (calculating a hypothetical minority-marketable value and then discounting for lack of marketability), but it is not certain that this would be a terribly valid approach, and the implication that it is minority-marketable could lead to other incorrect inferences.

General unattractiveness derives from onerous conditions associated with the ownership position, or uncaptured conditions involving the Property. A postulated short hold that results from an ability to force sale through a partition action would certainly require an increased risk premium for the likelihood of a lawsuit. The case does not contemplate forcing sale, but it is one of the options available to the interest holder if the expected hold does not work out, and another small adjustment is warranted.

Adjusted equity risk premium	12.1%
Holding period uncertainty	1.5%
Due diligence	0.5%
Business/management risk	1.0%
General unattractiveness	1.0%
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Investor's required rate	16.1%

*Sensitivity* of rate adjustments should be reported so the reader can better understand the quantitative effect of these adjustments. For the case, a change of 1% in the yield rate changes the concluded value by 3.5%.

Knowing the entity- and investor-level yields is not only useful for income methods, but can be used as a point of comparison in other methods. For now, the income method continues...

## **Income Methods - Discounted Future Returns and Present Value Models**

The value of any undivided interest is ultimately the present value of its future cash flows. Discounted future returns is a conventional valuation process, which has great power to accommodate irregularities of many types, including uneven cash flows, non-pro rata splits, as well as providing flexibility in dealing with explicit time periods. A difficulty for real property interests is the need to forecast operating line items, which are not always the same as presented in the real estate appraisal. Nonetheless, a nice discounted cash flow model in the real estate appraisal is a great starting point for fractional interest analysis. Add equity-level line items, including the projected NAV balance sheet and entity operations. The foregoing rate buildup can accommodate all levels of value, and as we saw above, the equity level DCF can be useful in extracting yield (IRR), cash flow and value growth for use in other discounting models.

The ability to just plug in risk adjustments, and accommodate multiple holding periods, allows incorporation of risks and underlying fact patterns from multiple tiers.<sup>11</sup>

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<sup>11</sup>Dennis A. Webb, ASA, MAI, MRICS and Lari B. Masten, MSA, CPA/ABV, CVA "A Methodology for

A present value model can also be used. This simplified method requires stable operations and constant growth (or at least a known growth pattern), and is identical to Chris Mercer's QMDM. It can produce the discount directly if NAV is set at \$1, then is grown over the holding period at the value growth rate. Cash flows can also be increased at the cash flow growth rate. The difference between the net present value and \$1 is the discount from NAV. The discounted future returns model produces the interest value directly, an effective discount from NAV would need to be separately calculated. The calculations are straightforward computations, and not detailed in this paper.

### Embedded assumptions and limitations

The QMDM has been criticized by many, and for various reasons, including an inability to predict the restricted stock discounts. One invalid criticism is its "excessive" sensitivity to inputs. It can only reveal the sensitivities inherent in the fact patterns, so if small changes in growth or yield rates produce large changes in discount, then maybe the discount *is* sensitive to these elements, and close attentions to facts & circumstances are indicated.

This is simply a present value model, and if applied as we have been applying such models for a long time, why would it be a problem? One issue might be that it is an investment model, with rates developed from observed investor behavior. How can we expect to model a very long hold (say, >10 years) if our rates are developed from market data for expected holds of 5-10 years at the most? How does the market discount distant events? The answer seems to be much less than one might think. Would the asset rates apply for long periods? How would we know? Model integrity can be preserved by using periods of 10 years or less, unless a special effort is made to develop support for long-term discounting.

What about short periods (<1-2 years)? Do asset *investment* rates apply? Isn't a short-term expected hold better conceptualized as a hedge or option? Short-term discounts on this basis are greater than an investment model would indicate, and are again nonlinear.

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Valuing Tiered Entities," *Journal of Business Valuation and Economic Loss Analysis* (Berkeley Electronic Press, Jan. 2006).

## Income Methods – Partition Time and Cost

The original “cost-to-partition” label has suggested that this is a cost-based method, but it is really an income method, based on a specific holding and reversion scenario. The only differences with the present value or DCF models we have been discussing is that a) the holding period is a function of a negotiating, litigation and sale time, b) direct costs must be included, c) risk is (typically) increased, and d) physical division of the property must be considered. The process is somewhat complex, and requires numerous estimates or opinions regarding costs, time and risk.

1. Consider a negotiation period prior to the lawsuit based on facts & circumstances of the case. Estimate costs and time for the lawsuit up to judgment. Costs and time vary with likelihood of challenge and counter suits, complexity (greater for partition-in-kind), and number and geographic location of cotenants and witnesses.
1. Decide whether the property is partitionable; if so, develop the partition process. Develop a plan for equitable division of the property, analyze cost and time associated with division (this process would occur after judgment)
2. Estimate additional costs after judgment for partitionable properties. The value of the resulting property will presumably be pro-rata share of value as decided by the court. (Issues of any changes in highest and best use, as well as particulars of the property, should have been handled by appraisers as part of the action.)
3. Estimate additional costs after judgment for non partitionable properties; usually includes costs of sale.
4. Prepare a discounted cash flow model:
  - Project or forecast cash flows from the property over the period.
  - Reversion is the expected sale price for the resulting property (court-ordered sale), or the market value of the divided portion received (partition-in-kind).  
  
The gross sale price is usually reduced by costs of sale, including commissions and transaction costs.
  - Assign calendar times to the cash flow elements.
5. Develop one or more yield (discount) rates.
  - Rates should reflect risk and uncertainty for at least two cash flow elements: 1) Real property (NOI and reversion) and 2) process-related costs. (More divisions can be made, if NOI and reversion, or the various costs, carry markedly different uncertainty/risk.)

- Risk elements considered earlier in the income approach build-up may be applied as-is, or there may be some differences, depending on the holding period and the circumstances of the partition scenario. A few examples:

The holding period was uncertain to some extent, but now the possible holding period range is much reduced. On the other hand, the model now covers a shorter period, and uncertainty may be a greater in proportion to the shorter period.

The potential for a lawsuit was added as an unattractive condition—*the partition scenario is a lawsuit*; removing “potential” as a qualifier may require an increase in yield. What yield would the market require when participation in a lawsuit is a condition of realizing the terminal value of the investment, particularly in relationship to alternatives?

- Costs may be discounted using a borrowing rate or any other appropriate method. The rate selected should reflect the likelihood of the costs being incurred. Would the buyer consider only maximum costs?
  - However, returns, including value reversion at the end of the process, must be discounted at a rate that will probably exceed the investor’s required rate. Cases showing real estate rates used for discounting partition actions grossly overvalue the interests for that reason.
6. The value of the interest by the cost-to-partition method is calculated as the present value of the subject interest's share of cash flows and reversion.

## Conclusions

- Partition action is not a certainty.

Economic feasibility depends on the size of the interest and complexity of the case, maybe resources of the other cotenants, property facts & circumstances.

- Risk is very often understated.

For method to be valid, it has to be a scenario considered by the hypothetical buyer of the interest, otherwise it is not a market-related method. This implies a willingness to enter a lawsuit, not generally undertaken lightly, or for free.

- Facts may show that a premature sale, such as a court-ordered sale in a declining market, or just before renewal of a major lease, are very disadvantageous to all parties.
- Partition-in-kind may be preferred, but is not always equitable, and may diminish the total property value, or changes its highest & best use.

## Comparative Company – Public Limited Partnerships

Public limited partnership interests are illiquid in the same sense as a private interest, in that the holder cannot get at its pro rata NAV. However, it is marketable. The price discount is viewed as largely impaired control—the inability to get at pro rata NAV.

The Partnership Profiles public limited partnership dataset has been used by valuers to develop discounts for lack of control (DLOC) for many years, and is arguably an investment model for generalized market holds, as postulated in the annual Rate of Return studies. Extending the original data to include one- and two-year estimated holds seems to work. But are such short holds still modeling an investment? Partnership Profiles has characterized buyer motivations as arbitrage, in its 2000 study of liquidating<sup>12</sup> partnerships (the “Liquidating Study”).<sup>13</sup> The Study includes transactions from various periods between 1996 and 2000, where liquidation at a particular date was expected.<sup>14</sup> For short holds, this is a near-perfect market representation.

The study includes trades in 70 partnerships, which took place after a liquidation plan was announced, and before sale of its remaining assets were completed. The anticipated holding periods varied from almost zero to 16 months. The range of holds from the 10th to 90th percentiles is approximately 1 to 7 months, and the range of discounts 4 to 29%. The medians are 2-3 months and 17%. There is little correlation for these short periods; the calculated correlation coefficient ( $R$ )<sup>15</sup> of 0.156 indicates that hold explains about 2.4% of the variation in the discount, no doubt because many influences could have a large effect on price over such a short period. Historic and current dividend yield does have an influence, interestingly. (Partnership Profiles indicates that distributing partnerships in this study averaged a 16% discount, while non distributing averaged 22%.)

This Study demonstrates that the price of liquidity (in terms of getting at pro rata NAV, or forecast distribution proceeds in the case of the Study) is much greater over a short period than an investment model would suggest—a 15% discount for a 6-month hold is an annual yield of 33%; a 20% discount would yield 45% (absent any distributions). The unaccounted influences appear to be less of a problem than for, say, restricted stocks, since the coefficient of variance for these data ( $CV$ ) is only 0.59.<sup>16</sup>

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<sup>12</sup>The word “liquidating” is used here with respect to the sale of partnership assets, wind-up of operations, and distribution of proceeds. It contemplates an orderly sale of assets, less transaction costs, but does not consider forced sale conditions which might result in *liquidation value*, not *market value*, for the real estate.

<sup>13</sup>“Secondary Market Buyers Playing Arbitrage,” *Partnership Spectrum*, (March/April 2000): 1-6.

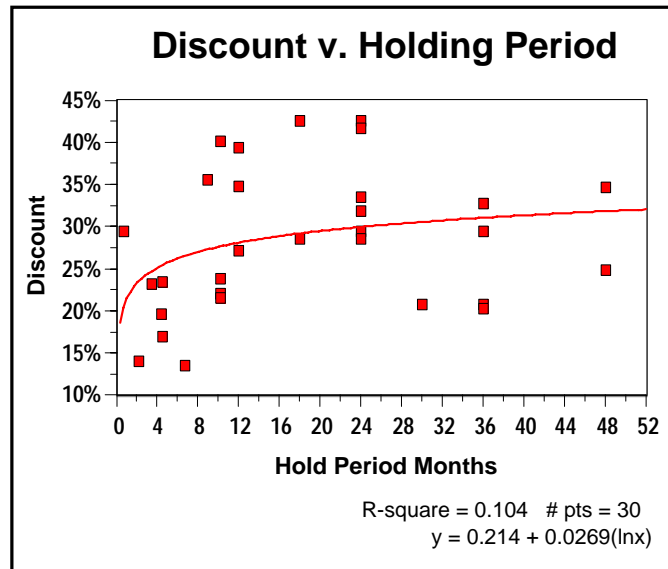
<sup>14</sup>The transaction date is selected differently, since proximity to the liquidation announcement is the principal criterion, not publication of the previous year’s 10K, as for the Re-Sale Discount Study.

<sup>15</sup>The correlation coefficient ( $R$ ) describes the strength of an association between variables—the extent to which the value of one variable can be predicted by the value of the other. The square of the size of the correlation coefficient is the fraction of the variance of the one variable that can be explained from the variance of the other variable.

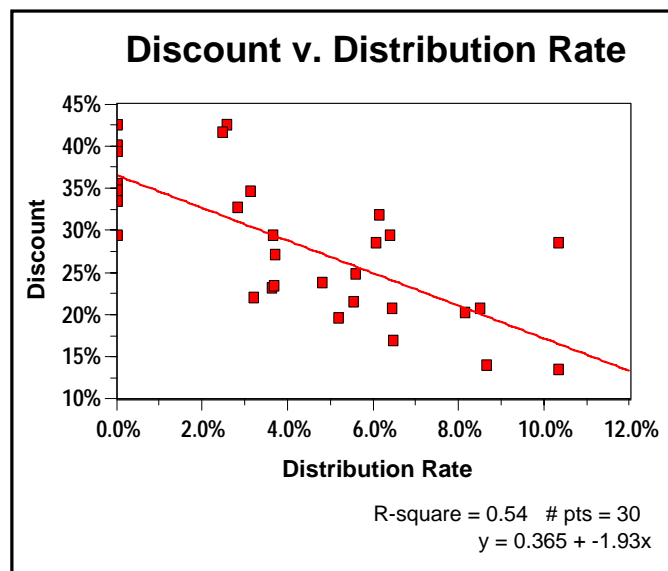
<sup>16</sup> $CV = \text{Standard Deviation} / \text{Mean}$ .  $CV$  is a measure of data dispersion, and a smaller  $CV$  indicates that the

## Extended Dataset

In order to have a more usable data set, we rejected some Liquidating Study transactions because distribution rates could not be obtained, there were three months or less between the announcement and the actual sale (it was not clear that the buyer would have had adequate information), and also some of the more obvious outliers. We added in data from the annually published discount studies, and ended up with 30 transaction sets (groups of transactions in a single partnership for a particular year) from 26 different partnerships. Of these 30, all but seven pay regular distributions, which range from 2.5 to 10.3% (measured with respect to net asset value, not the transaction price). The remaining time, before expected liquidation, ranges from 2-3 months to four years. All property types are represented.



Discounts range from 13% to 42%, with a median of 28.7%. The median holding period was 15 months. The graph the discount as a function a holding period. The relationship is clearly significant, and is fit best by a logarithmic curve. The second curve shows the discount as a function of the current distribution rate. That the influence of distributions has greater significance is perhaps surprising, since a short hold would suggest that the pro rata share of distributable proceeds on wind up would be by far more important in the buyer's thinking.



Using estimated holding periods has improved the usefulness of a huge database of partnership trading data, as noted in the earlier discussion of the Rate of Return Study. Extending that database to include very short holds provides both evidence of differing market behaviors for short-term discounts, and a decent method of quantifying them.

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data is more tightly grouped. The annualized rate is calculated as  $12 \times$  the monthly compounded rate.

The original study is available from Partnership Profiles, and a survey of their discount study data will turn up some partnerships for which longer holds were expected. A simple multilinear regression can provide a discount indication as a function of hold time and distribution rate. For example, a multilinear regression using the data from the graphs above, but including both dividend yield and holding period (log transformation) produces decent, usable results, with an  $R^2$  of 0.71; dividend yield is the most significant, with a t-statistic of 7.08, and the hold period t-statistic is 3.92. P-values are very low.

### **Accounting for Control and Liquidity Differences**

The public limited partnership interests typically have no effective control, so adjustment must be made to any public partnership- derived discount for the market effect of the subject's superior control. One method of estimating a reasonable adjustment range would be to observe discounts attributable to nonvoting stock, which is priced roughly 0% to 8% under otherwise identical voting stock. The adjustment would depend on whether the subject interest can exert any meaningful control, given the likelihood that much of the control prerogative may already be captured in the short-hold conclusion.

The public limited partnership interests also contain a measure of liquidity, which requires no adjustment for very short holds. For those cases with expected holds longer than, say, 2-3 months, the ability to sell the interest should have value, although such an early sale would only yield the discounted fair market value of the interest, not the pro rata equity that is expected to be received at the end of the hold.

Holding company applications for very short-term holds are rare, but applications for 1-2 or 3 year holds are not—they are typical of cotenancies or general partner positions where forcing sale is a real option (permissible, enough value in the interest to make it financial feasible if opposed).

The near-term liquidating partnerships method is, again, an imprecise tool. There are, no doubt, many more such transactions now that a great many partnerships have wound up their operations in one form or another. A more detailed, published study would make the data more readily available. Objections are sometimes raised to the use of partnership data to support cotenancy, or even general partnership, discounts. However, it should be clear that a careful interpretation of the case facts & circumstances, leading to a well-supporting holding period expectation, should allow application of these data. They are yet another interpretation of market behavior, and helpful support for the discount conclusion.

## Options Methods

Articles by Chaffe<sup>17</sup>, Griffith,<sup>18</sup> and now many others examine the cost of put options as a method for measuring discounts for lack of liquidity. They often refer to the Black-Scholes option pricing model, which provides strong theoretical support for the observation that risk (here interpreted as the expected *volatility* of a security interest) has a direct effect on the magnitude of its discount. They model an investor position where the unrestricted value of the underlying security is realized after a holding period, a reasonable mirror of the cotenancy or general partner position (under the premise set forth in this paper).

Inputs are the risk-free rate, the remaining term of the option, and the volatility<sup>19</sup> of the underlying security. Present value of any dividend stream can be accommodated with simple additions to the formula. Historic volatility can be measured for actively-traded issues, but a principal shortcoming of applying this method to privately-held companies is that volatility cannot be directly measured if the interests are not traded. After all, what is the volatility of a single fixed asset with its ownership structure? Mercer raises this as an objection to using Black-Scholes,<sup>20</sup> along with others.

Given the obvious lack of precision with our available methods for analyzing discounts, why not use publicly-traded securities as volatility proxies? Or even an approximation based on yield rates, since “yield and volatility are clearly connected, although there is probably no theoretical relationship”<sup>21</sup> For example, a low-risk situation may exhibit an effective volatility of 0.2, moderate risk 0.3, and higher risk 0.5. One difficulty in obtaining volatilities for single-asset holding situations is that listed companies are far more diverse, so volatility in share pricing is not necessarily reflective of price volatility for a single asset. A better method is to develop a relationship between yield and volatility generally, and then apply it to a yield rate that can be developed for the specific property. A long-term triple-net lease to a major corporation would, obviously, have less volatility than an office property in an overbuilt market with tenants having short remaining lease terms and above-market rents. Any risk factors attributable to the facts of the case, as noted in the earlier rate buildup. Studies of yield vs. volatility are underway, and will hopefully provide some degree of rigor that can be readily used in this model.

The option itself is a marketable security, which provides some possibility for its holder to take advantage of changing conditions. Whether this would be meaningful for a

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<sup>17</sup>David B. H. Chaffe III, “Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations,” *Business Valuation Review* (December 1993): 182-188.

<sup>18</sup>Reynolds Griffith, Ph.D., “Valuation of Restricted Stocks: An Option Theory Approach,” *ASA Valuation* (February 1988): 96-102.

<sup>19</sup>The standard deviation of annualized returns; see references.

<sup>20</sup>See Mercer, pages 404-414.

<sup>21</sup>Dwight Grant, Ph.D., “Relative Pricing of Complex Securities,” *Current Topics in Business Valuation* (ASA Los Angeles, May 9, 2007)

privately-held interest, given the goal of selling at pro rata NAV rather than FMV, is questionable, as noted earlier. A limitation of the model is that, while accounting for avoiding unexpected loss during the holding period, it does not allow for the possibility of realizing unexpected gain. It is a stop-loss model that does not capture any upside potential. Thus, it should generally indicate an upper limit to the discount.

A further limitation is that this is a short-term model, at least as far as the market for our fractional interest is concerned. Eliminating price risk during a period of impaired liquidity is a market-observed behavior for listed options, and the hedging objective reasonably characterizes a short-period position. A longer hold is most reasonably modeled as an investment, not as a hedge, limiting the appropriateness of an option model. (This is a market-related observation, *not* an informed opinion about the validity of option models for long periods.)

Black-Scholes is an empirical formula, designed to model market behavior. It seems reasonable that this behavior is more *speculative* in its nature than behavior observed in investment markets, where yields over longer periods are the basis for decision-making. It would be more aligned with short-term behavior. In spite of the obviously loose connection with the volatility term, options models can provide useful value indications for short-term holds, and give use one more slant on market behavior.

### **Longstaff Look-Back Put**

The Black-Scholes model presents a set of limitations that are hard to overcome. A more elegant and directly-usable model may be the Longstaff Look-Back Put,<sup>22</sup> as brought forward by Ashok Abbot.<sup>23</sup> The model measures price risk during the holding period, also defined as the period of liquidation.

It is backward-looking, and effectively isolates the maximum cost of illiquidity. Its mathematics are based on Black-Scholes; variables include the difference between the maximum price and the ending selling price, volatility and the holding period. It assumes normally distributed returns and perfect timing. The “look-back” characteristic is based on your receipt of the maximum price at the end of the holding period. It seems that this model will improve the utility of Black-Scholes for our purpose, but further work needs to be done (particularly on volatility estimates).

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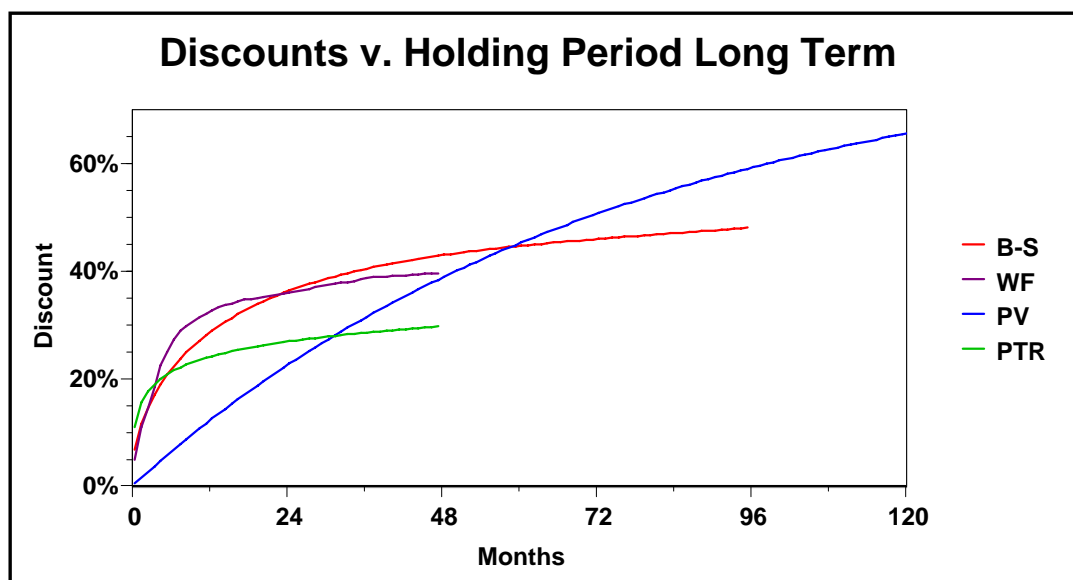
<sup>22</sup> Francis A. Longstaff, “How much Can marketability Affect Security Values?,” *The Journal of Finance* (December, 1995): 1767- 1774.

<sup>23</sup> Ashok B. Abbot, Ph.D., “Discount for Lack of Marketability: An Empirical Analysis,” *Business Valuation Review* (December, 2003): 1720179. The holding period notion is developed in detail in Dr. Abbot’s “Estimating the Holding Period for Listed Securities,” *Valuation Strategies*, (September/October 2004): 4-98, 42.

## Graphic Illustration for Methods and Time

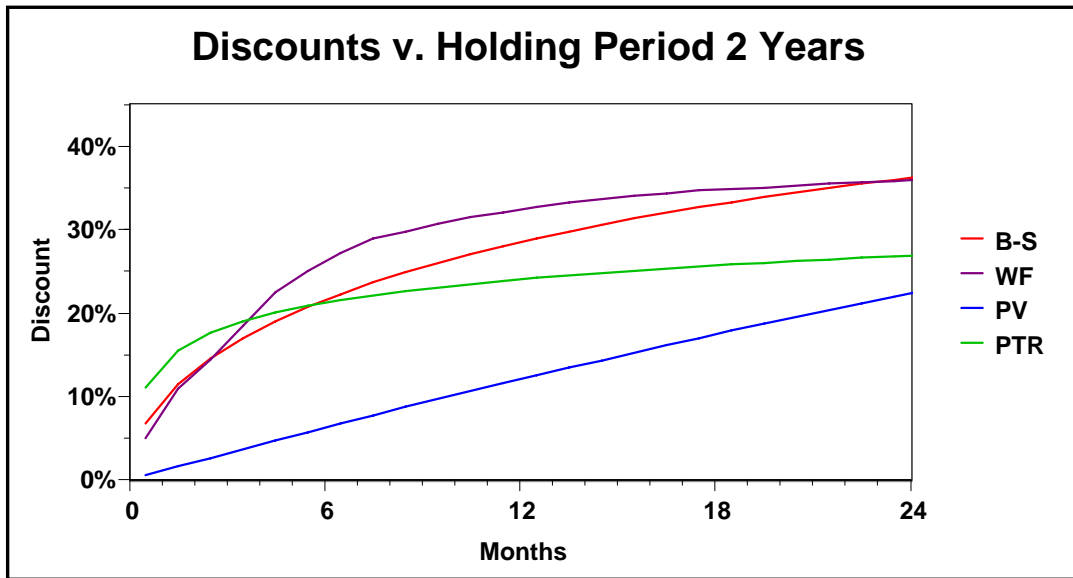
It appears that the methods just reviewed, present value, options, short-term LP trades and restricted stock, are valid for certain holding periods only. The following curves illustrate discounts as a function of time. The first shows a 10-year period. The present value (PV) model is clearly describing a different market than the other models below 24 months. After that it is a conventional income approach, that can be evaluated based on established principles. However, whether it applies to very long periods is questionable, as noted earlier.

The Black-Scholes (B-S) calculations were only carried out to eight years. If this model describes an upper limit to the discount, then it either does not apply for long periods, or its ability to describe the market breaks down. The restricted stock curves developed by Will Frazier (WF) are carried out only four years, and the data for the short-term liquidating partnerships (PTR) do not include any data for longer expected holds.



The next set of curves (next page) shows the 2-year period, where the differences are more obvious. There is reasonably good conformance between the shape of the curves for all but the present value model. The notion that a present value model does not describe the market for a short-term hold is supported by three different empirical functions.

The purpose of this presentation is to show the shape of the curves for each method, particularly at the lower end of the time scale. They were generated from the same input data (except for the restricted stock curve, which was taken directly from the Valuation Strategies article), but their relative positions will change depending on the specific case.



## ***Conclusions***

- **Facts & circumstances drive the entire value analysis and produce holding period and risk measures suitable for quantitative models**

*Be able to identify asset-related facts & circumstances that influence discounts.*

The nature of the asset(s) underlies the fact patterns.

Risk and holding period are fundamental interpretations of the facts.

The valuation's starting point is the asset level of value.

- **Models interpret different market behaviors, and vary in their appropriateness and predictive ability**

*Understand characteristics of discounting models and how to connect them with case facts & circumstances.*

### ***Income (present value) models***

- Discounted cash flow
- Constant growth/present value
- Partition time and cost (“cost-to-partition”)

### ***Options models***

- Black-Scholes

### ***Comparative company methods***

- Direct sales of fractional interests
- Studies of direct sales
- Restricted stock studies
- Short-term liquidating partnerships

- **Variety of models allows wide range of fact sets to be viewed from different “angles”**

*Interpret and reconcile the models' discount indications.*

- **Tools are sufficient for reliable analysis of all asset holding entities**

*Take home commonsense solutions to frustrating valuation problems.*

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